



Transducer Array Interaction Modeling Using Variational Principles

Presented at the 127th Meeting of the
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Cambridge, Massachusetts

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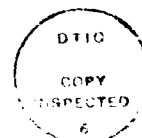
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PREFACE

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A handwritten signature in black ink, reading "B. F. Cole". The signature is written in a cursive style with a large, stylized "C" at the end.

B. F. Cole
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TRANSDUCER ARRAY INTERACTION MODELING USING VARIATIONAL PRINCIPLES

INTRODUCTION

This document will detail a method for calculating the radiation interaction effects between active transducers in an array, commonly known as the radiation impedance, using variational methods. A brief review of previous treatments of the radiation impedance for single mode and multimode transducers will be given. These interaction treatments will be modified, using the variational principle, to yield expressions for the self and mutual radiation impedance which do not require exact solutions for the surface pressure of the radiator. Specific calculations of the self and mutual impedances for baffled pistons and flexural disks will be presented and compared with previous theoretical treatments. The extension of these variational expressions to flexensional transducers will be briefly discussed.

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RADIATION IMPEDANCE

- SINGLE MODE PLANAR TRANSDUCERS (PRITCHARD, 1960)
- TRANSDUCER MOTION DESCRIBED BY FIXED VELOCITY DISTRIBUTION
- FORCE ON n th RADIATOR DUE TO PRESENCE OF N RADIATORS IN ARRAY

$$f_n = \sum_{m=1}^N Z_{nm} v_m \quad Z_{nm} = R_{nm} + jX_{nm}$$

WHERE Z_{nm} IS THE MUTUAL RADIATION IMPEDANCE BETWEEN m AND n

- NET RADIATION IMPEDANCE ON n th RADIATOR

$$Z_n = \frac{f_n}{v_n} = \sum_{m=1}^N Z_{nm} \frac{v_m}{v_n}$$

- RADIATION IMPEDANCES FOR TWO PISTONS $((ka)^2 \ll 1, (a/d) \ll 1)$

$$\frac{Z_{11}}{\rho c a^2} = \left[1 - \frac{J_1(2ka)}{ka} \right] + j \frac{H_1(2ka)}{ka} \quad Z_{12} = R_{11} \left[\frac{\sin kd}{kd} + j \frac{\cos kd}{kd} \right]$$

FIGURE 1

The classic theoretical treatment of mutual interaction is that of R.L. Pritchard ["Mutual Acoustic Impedance Between Radiators in an Infinite Rigid Plane," J. Acoust. Soc. Am. 32, pp.730-737 (1960)] for circular pistons in an infinite rigid baffle. Because the pistons are assumed to be rigid as well, their motion is defined by a fixed, constant velocity distribution. The total force on a specific piston in the array can then be described as the sum of the mutual impedances between each element multiplied by their velocities. The total radiation impedance on the radiator is that force divided by the velocity of the radiator. C.J. Bouwkamp ["A Contribution to the Theory of Acoustic Radiation," Phil. Res. Rep. 1, pp. 262-264 (1946)] determined that the radiation impedance for a radiator can be calculated by integrating the directivity pattern $K(\theta, \phi)$ of the radiator over real and imaginary values of θ . For two identical pistons, the directional characteristic is the product of K for a single radiator and K for two point elements. Carrying out this integration, Pritchard determined the self radiation impedance in terms of the Bessel function $J_1(2ka)$ and the Struve function $H_1(2ka)$, ka being the dimensionless radius of the piston. The mutual impedance, in its analytic form, was found to be a doubly infinite sum of products of Bessel functions and spherical Hankel functions. After taking limits for the size of the pistons $[(ka)^2 \ll 1]$ and for their separation distance kd [$kd \gg ka$], Pritchard arrived at his famous "rule": that the mutual impedance between radiators is simply the product of the self radiation resistance and simple sinusoidal functions which decrease as the distance between the radiators increases.

RADIATION IMPEDANCE

- **PRITCHARD MODEL FOR MUTUAL IMPEDANCE VALID ONLY FOR:**
 - **PLANAR ARRAYS IN INFINITE RIGID BAFFLE**
 - **SINGLE MODE OF OPERATION (DESCRIBED BY VELOCITY)**
 - **VERY LOW FREQUENCY or**
 - **SMALL RADIATOR SIZE and**
 - **LARGE INTER-RADIATOR SPACING**
 - **LOW FREQUENCY ARRAYS OF FLEXTENSIONAL TRANSDUCERS DO NOT USUALLY CONFORM TO THESE CHARACTERISTICS**
 - **PREVIOUS ANALYSES USING PRITCHARD'S RULE FOR ARRAYS OF FLEXTENSIONAL TRANSDUCERS HAVE SHOWN THAT ERRONEOUS BEAM PATTERNS MAY BE CALCULATED, PARTICULARLY WHEN THE ARRAY IS STEERED**
 - **NEED TO DEVELOP FORMALISM FOR CALCULATING RADIATION IMPEDANCES WHICH DOES NOT RELY ON PRITCHARD ASSUMPTIONS**
-

FIGURE 2

Unfortunately, this simple model is not a valid approximation for many types of arrays currently in use. The reason for this is that the assumptions used in deriving Pritchard's rule are not applicable to other types of transducers, particularly flexensionals, because their motion cannot be described by a single velocity. Furthermore, many arrays violate the assumption that the radiators are spaced far apart. It has been seen in previous analyses that arbitrarily using Pritchard's expression for the mutual impedance in the modeling of arrays of flexensional transducers leads to incorrect predictions for the beam pattern, especially when the array is steered. This is caused by the fact that, in a steered array, the velocity distributions on the radiators are no longer identical due to phasing. Because of these inadequacies, it is necessary to develop a formulation for predicting the radiation impedance which does not depend on Pritchard's assumptions for radiator type, size and separation.

RADIATION IMPEDANCE

• MULTIPLE MODE TRANSDUCERS (SHERMAN, 1970)

• TRANSDUCER VELOCITY DESCRIBED BY IN-VACUO MODE EXPANSION

$$v(\vec{r}_i) = \sum_{n=0}^{\infty} V_{ni} \eta_n(\vec{r}_i)$$

• SURFACE PRESSURES CAN BE EXPANDED USING THE SAME COEFFICIENTS

$$p(\vec{r}_i) = \sum_{m=1}^N \sum_{n=0}^{\infty} V_{ni} p_{ni}(\vec{r}_i)$$

• POWER RADIATED BY THE ARRAY

$$\begin{aligned} P &= \frac{1}{2} \operatorname{Re} \iint_S v'(\vec{x}_s) p(\vec{x}_s) dS \\ &= \frac{1}{2} \operatorname{Re} \sum_{j=1}^N \sum_{m=0}^{\infty} \sum_{i=1}^N \sum_{n=0}^{\infty} V_{mj} V_{ni} Z_{mnij} \end{aligned}$$

• Z_{mnij} IS THE MODAL RADIATION IMPEDANCE

$$Z_{mnij} = \frac{1}{V_{ni} V_{mj}} \iint_{S_j} V_{ni} p_{ni}(\vec{r}_i) V_{mj}^* \eta_m(\vec{r}_i) dS_j$$

FIGURE 3

One such formulation, for transducers capable of radiating in multiple modes, was developed by C.H. Sherman ["General Transducer Array Analysis," Scientific Rept. No. 6, Cont. N00014-67-C-0424, Parke Mathematical Laboratories (1970)]. Consider an array of N radiators. The surface velocity of the j th radiator may be expressed, as in many standard treatments, as an expansion in its *in-vacuo* vibration modes. The coefficients V_{nj} may also be used to expand the pressure on the j th radiator. This expansion includes contributions from all the other N radiators in the array. The total power radiated by the array is an integral of the surface velocity multiplied by the pressure over the radiating area. Summing over each of the N surfaces and substituting in the pressure and velocity expansions allows the power to be expressed as a double sum over the radiators and a double sum over the modes. The factor Z_{mnij} is the mutual radiation impedance between the n th pressure mode of the i th radiator and the m th velocity mode of the j th radiator, evaluated over the surface of the j th radiator. The procedure outlined is completely general and does not depend on exactly how one determines the radiated power in terms of the velocity coefficients. The method for determining the pressure function $p_{ni}(\vec{r}_i)$ has also not been explicitly specified, nor is an exact solution for the velocity coefficients required. Therefore, this expression for determining the radiation impedance is suitable for variational analysis.

VARIATIONAL CALCULATION OF RADIATION IMPEDANCE

- ASSUME EACH RADIATOR IS VIBRATING IN ONE PARTICULAR MODE
- ASSUME FOURIER EXPANSION FOR PRESSURE

$$p(r, \theta) = \frac{1}{2} \rho c^2 \sum_{m=-\infty}^{\infty} \sum_{j=1}^L P_{mj} \Psi_{mj}(r) e^{jm\theta}$$

- VARIATIONAL EXPRESSION FOR RADIATED POWER (PIERCE, 1987)

$$P = \frac{\rho c^2 a}{2k} \operatorname{Re} J[p]$$

WHERE

$$J[p] = -\frac{i}{8\pi} \sum_{m=-\infty}^{\infty} \sum_{j=1}^L \sum_{l=1}^L P_{mj} P_{lj} A_{mj} - \frac{ka}{2} \sum_{m=-\infty}^{\infty} \sum_{j=1}^L \sum_{l=1}^N P_{mj} v_{lj} B_{mj} + \frac{(ka)^2}{4\pi} \sum_{l=1}^N \sum_{j=1}^N v_{lj} v_{mj} K_{lj}$$

- $J[p]$ IS STATIONARY WITH RESPECT TO VARIATIONS IN PRESSURE

$$\delta J = \sum_{m=-\infty}^{\infty} \sum_{j=1}^L \frac{\partial J}{\partial P_{mj}} \delta P_{mj} = 0$$

FIGURE 4

To implement a variational treatment for the modal radiation impedance, we first assume that each element in the array is vibrating in a single mode, which will be marked by the coefficient v_m . Thus the surface velocity of each radiator is prescribed exactly. The surface pressure in polar coordinates is approximated as a Fourier series in the angular variable θ , while the radial dependence is represented by an approximate trial function Ψ_{mj} which will incorporate as many of the physical characteristics of the exact solution as possible. A variational formulation of the radiated power was developed by A.D. Pierce ["Stationary Variational Expressions for Radiated and Scattered Acoustic Power and Related Quantities," IEEE J. Ocean. Eng. OE-12, pp. 404-411 (1987)] for radiators with prescribed surface velocities. With our expressions for the velocity and surface pressure, the power is written in terms of a functional $J[p]$, which has summations over the number of pressure trial functions L , the number of radiators N , and the harmonics. From the form of $J[p]$, each pressure mode is related to every other pressure mode through the matrix $[A]$ and to the surface velocity modes of all the radiators via the matrix $[B]$. The most important characteristic of $J[p]$ is that, because it is variational in nature, it is stationary with respect to variations in the pressure. This means that we can differentiate $J[p]$ with respect to the pressure expansion coefficients P_{mj} . Because the variation δP_{mj} is arbitrary, this variation leads to a set of simultaneous linear equations for the pressure expansion coefficients. In theory this set of equations is infinite due to the harmonics; however, in practice we truncate the harmonic summation to $2M+1$ terms.

VARIATIONAL CALCULATION OF RADIATION IMPEDANCE

- SOLVING THIS SET OF SIMULTANEOUS EQUATIONS RELATES PRESSURE COEFFICIENTS TO RADIATOR VELOCITIES

$$\{P_m\} - i2\pi(ka)[A_m]^{-1}[B_m]\{v\} - i2\pi(ka)[G_m]\{v\}$$

- RE-EXPRESS $J[p]$ IN TERMS OF VELOCITIES

$$J[p] - \pi(ka)^2 \{v\}^T \left[\sum_{m=0}^{\infty} (2 - \delta_{0m}) \left(\frac{1}{2} [G_m]^T [A_m] [G_m] - i [G_m]^T [A_m] \right) + \frac{1}{4\pi^2} [K] \right] \{v\} \\ - \pi(ka)^2 \{v\}^T [J] \{v\}$$

- VARIATIONAL EXPRESSION FOR RADIATED POWER NOW CONFORMS TO SHERMAN'S EXPRESSION
- RADIATION IMPEDANCES NOW CAN BE ASSOCIATED WITH ELEMENTS OF THE $N \times N$ MATRIX $[J]$
- DIAGONAL ELEMENTS GIVE MODAL SELF RADIATION IMPEDANCES
- OFF-DIAGONAL ELEMENTS GIVE MODAL MUTUAL RADIATION IMPEDANCES

FIGURE 5

Due to the form of $J[p]$, each of the pressure harmonics is decoupled, simplifying the solution somewhat. The matrix $[A]$ can be separated into $L \times L$ component matrices $[A_m]$, while the matrix $[B]$ can be divided into $N \times L$ submatrices $[B_m]$. Solving the system of equations arising from the pressure variation allows us to relate the set of L pressure expansion coefficients $\{P_m\}$ for the m th harmonic to the set of N velocities of the radiators $\{v\}$, thereby expressing the pressures in Sherman's form. Substituting for the pressure coefficients in the functional $J[p]$ now yields an expression solely in terms of the velocities of the radiators, where the matrix $[J]$ has the dimensions $N \times N$. Therefore, the variational expression for the radiated power now conforms to that of Sherman, and therefore we can determine what the modal radiation impedances are from the elements of the radiated power matrix $[J]$. The diagonal elements yield the self radiation impedances, since they correspond to contributions to the radiated power where the velocity coefficients are the same. The off diagonal elements give the mutual radiation impedances. As you might expect, the matrix $[J]$ is symmetric.

GEOMETRY FOR RADIATION IMPEDANCE CALCULATION

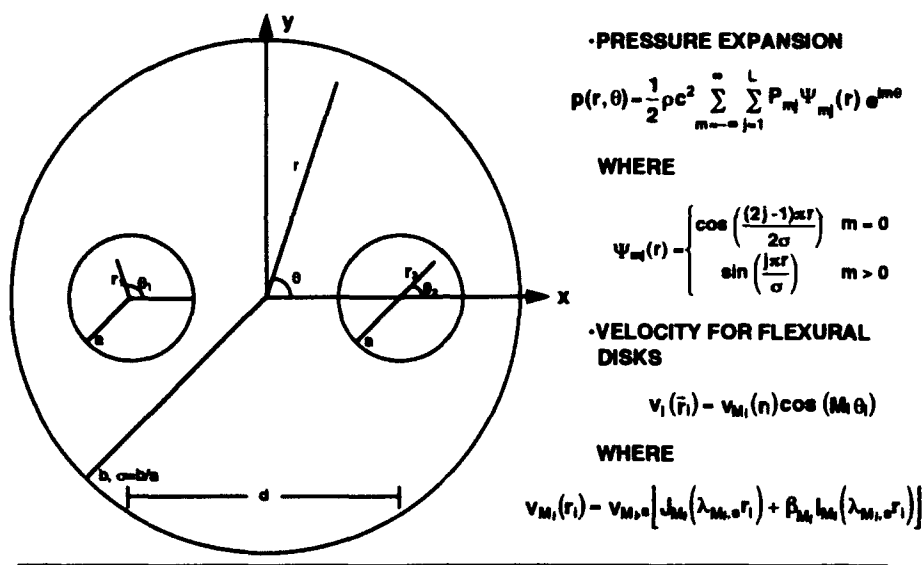


FIGURE 6

To test this hypothesis, calculations for circular pistons and flexural disks in an infinite rigid baffle were made. The radiators have a radius a , and the motions of each can be described in a local coordinate system (r_i, θ_i) with origins at the radiator center. The surface velocities of the radiators are either uniform (in the piston case) or represented as modes of a simply supported elastic plate, which most closely conforms to the ideal edge condition for a flexural disk in a baffle. The coefficients $\lambda_{M_i,s}$ and β_{M_i} are determined from the simply supported boundary conditions [A.W. Leissa, "Vibrations of Plates," NASA SP-160, Washington, D.C., 1969], where M_i denotes the number of nodal lines and s the number of nodal circles. The rigid baffle is assumed to be circular, with a radius $b \gg a$ to simulate an infinite baffle. This baffle defines a global coordinate system (r, θ) in which the pressure trial functions are defined, in essence over all space. The behavior of the surface pressure on the circular baffle must be included explicitly because the Helmholtz integral equation, from which the variational expression is derived, requires that the integrating surface be either closed or infinite and planar [J.H. Ginsberg, P.T. Chen and A.D. Pierce, "Analysis Using Variational Principles of the Surface Pressure and Displacement Along an Axisymmetrically Excited Disk in a Baffle," J. Acoust. Soc. Am. **88**, pp. 548-559 (1990)]. The radial dependence of the trial functions $\Psi_{mj}(r)$ is chosen such that the pressure functions become zero at the edge of the baffle and satisfy continuity conditions at the origin.

COMPARISON OF SELF RADIATION IMPEDANCE (PISTONS, 15 PRESSURE FUNCTIONS)

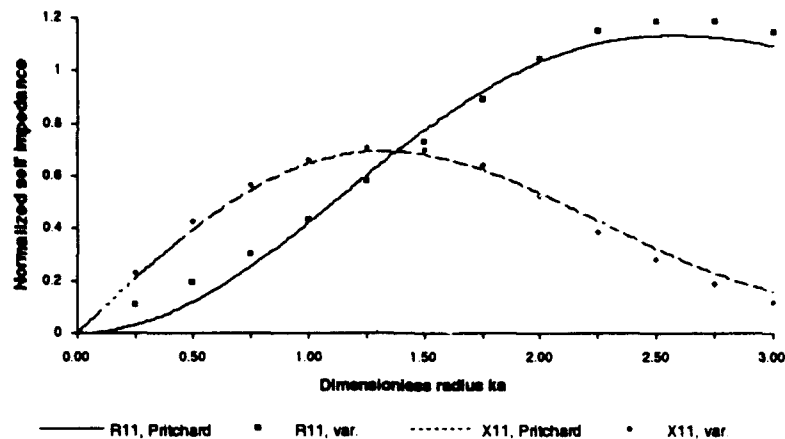


FIGURE 7

The variationally determined self radiation impedance for a piston radiator is compared here with closed form results from Pritchard. To expedite the calculation, it was assumed that the separation distance between the two radiators was zero, i.e., that they overlapped. Due to the axisymmetry of the surface velocity, only the $m=0$ harmonic is required in the pressure expansion. The agreement between the variational and exact solutions is very good for the most part, with poorer agreement as the dimensionless radius ka decreases. The reason for this poor agreement is that more pressure trial functions are required for smaller ka than for larger ka , due to the form of the pressure trial function and the manner in which the matrix $[A]$ is calculated. This is apparent because it appears that the radiation resistance, in the variational case, does not appear to be approaching zero as it should. The variational method is not very satisfactory for this case because it takes much longer to compute the result than using the analytical expression

COMPARISON OF SELF RADIATION IMPEDANCE
(SIMPLY SUPPORTED FLEXURAL DISK IN 0-0 MODE,
15 PRESSURE FUNCTIONS)

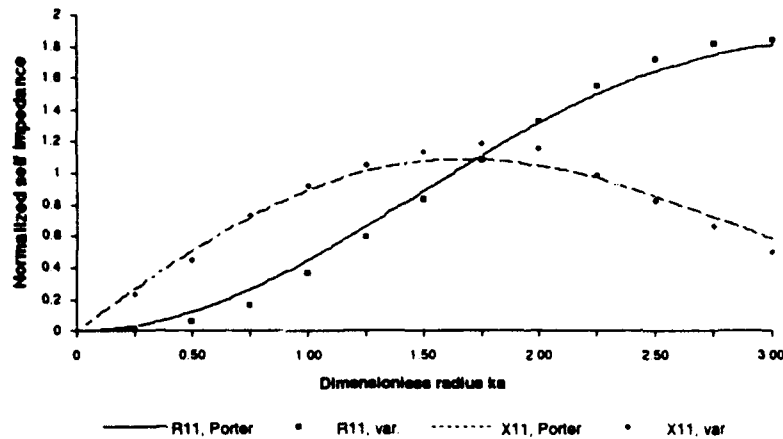


FIGURE 8

A similar result occurs for the lowest order ($M_1=0$, $s=0$) flexural disk mode. Self and mutual radiation impedances for flexural disks radiating in their fundamental mode were computed by D.T. Porter ["Self- and Mutual-Radiation Impedance and Beam Patterns for Flexural Disks in a Rigid Plane," J. Acoust. Soc. Am. 36, pp. 1154-1161 (1964)] based on an approximate expression for the surface velocity. The analytic results for the self radiation impedance were given in terms of zeroth and first order Bessel and Struve functions of argument $2ka$. As in the piston case, only the $m=0$ harmonic of the pressure expansion is required for the variational calculation, because the flexural disk is moving axisymmetrically. Again, the agreement between the variational method and the analytical result is quite good, with the largest discrepancy occurring for small values of ka and for the reactance. Unlike the piston case, it appears that the variational result does go to zero as ka approaches zero. This improved convergence is due to the shape of the velocity distribution for the flexural disk case. The surface velocity goes to zero at the edge of the flexural disk, which provides a built-in convergence factor in the calculations of the matrices [B] and [K].

**COMPARISON OF MUTUAL RADIATION IMPEDANCE
(PISTONS, $ka=1$, 15 PRESSURE FUNCTIONS, 5 HARMONICS)**

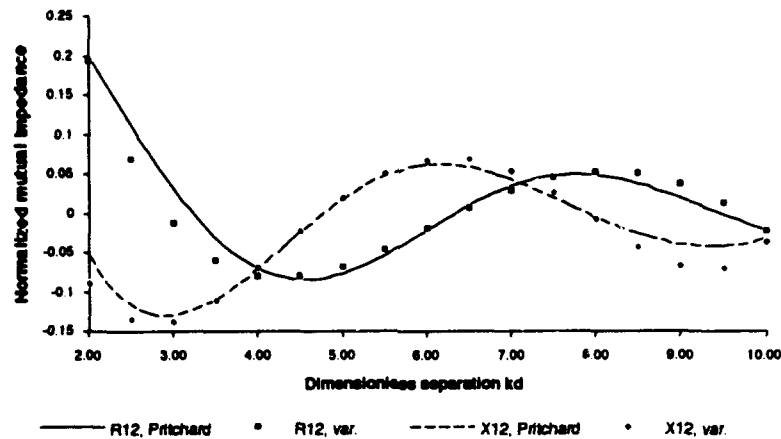


FIGURE 9

The mutual radiation impedance for two identical pistons, calculated variationally, is compared to Pritchard's analytical results for a range of values of separation distance kd . Recall that the mutual impedance, in its exact form, is a double infinite sum of products of Bessel functions of argument ka and spherical Hankel functions of argument kd . Since we now have two radiators present, the axisymmetry of the problem has been eliminated and now several azimuthal harmonics must be used. As might be expected, the further apart the radiators are, the fewer pressure expansion functions are required for convergence, and the better the agreement. As kd decreases, the agreement becomes worse, which is to be expected. The closer together the radiators are, the more pronounced the interaction effect, and hence more trial functions are required to model it well. We also note that, for larger values of kd , the agreement again deteriorates. This is because, as the separation between the radiators increases, the angle θ (in the global coordinate system) subtended by them decreases. Therefore, higher harmonics in the pressure expansion are required to match the behavior of the velocity distribution. A subject for future study would be to address the interplay between separation distance and number of trial functions and/or harmonics required for convergence. In addition, it must be noted that other trial functions for the radial pressure dependence, such as Bessel functions, could be used which satisfy the imposed boundary conditions; these alternative trial functions might improve the convergence of the variational approximation as well.

**COMPARISON OF MUTUAL RADIATION IMPEDANCE
(SIMPLY SUPPORTED FLEXURAL DISKS IN 0-0 MODE,
 $ka=1$, 15 PRESSURE FUNCTIONS, 5 HARMONICS)**

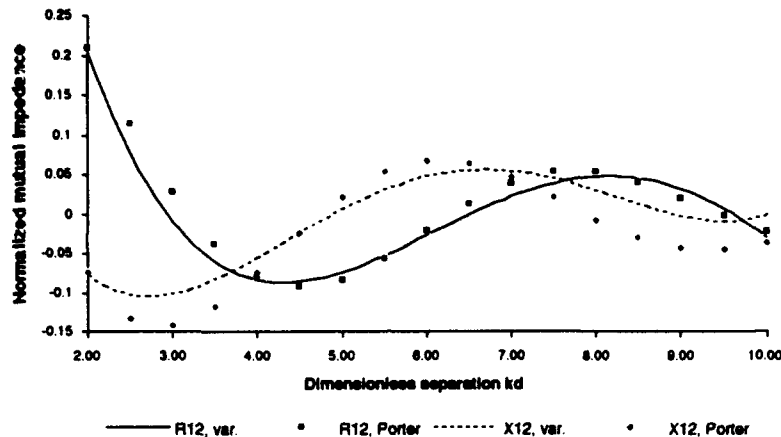


FIGURE 10

The mutual radiation impedance of two flexural disks in the fundamental mode is compared with Porter's analytical results. The analytical mutual impedance solution involves two infinite sums and four finite sums over Bessel functions of argument ka and spherical Hankel functions of argument kd . Again, the agreement is quite good, with smaller values of kd leading to larger discrepancies between the variational and exact values. Also, the deterioration in X_{12} due to neglecting the higher harmonics is demonstrated as well. It appears that the overall agreement for the flexural disk case is worse than for the piston case, a possible consequence of the more complicated surface velocity distribution, and hence more complicated forms for the $[B]$ and $[K]$ matrices. In particular for large kd , the variation in the surface velocity would require even more harmonics be included than in the piston case. The computational efficiency of the variational method is now comparable to that of the exact solution, where many terms in the infinite sums are required for the exact solution to converge. This shows that, while a variational calculation may not be desirable for the simpler situations for which exact solutions exist, it does begin to demonstrate some merit for more complicated situations. Also note that Pritchard's sinusoidal dependence on the separation distance appears to hold in this case, which is understandable because we are still dealing with a planar radiator in an infinite rigid baffle.

VARIATIONAL MODELING OF RADIATION IMPEDANCE-- CONCLUSIONS

- SHERMAN FORMULATION OF MUTUAL RADIATION IMPEDANCE ALLOWS FOR ANALYSIS OF MULTI-MODE TRANSDUCERS
 - VARIATIONAL FORMULATION FOR RADIATED POWER MAY BE RECAST INTO SHERMAN'S FORM, ALLOWING THE MODAL RADIATION IMPEDANCE TO BE CALCULATED VARIATIONALLY, WITHOUT NEEDING TO SOLVE FOR THE SURFACE PRESSURES EXACTLY
 - COMPARISONS OF VARIATIONAL RESULTS WITH EXACT CALCULATIONS FOR PISTONS AND FLEXURAL DISKS IN RIGID BAFFLES ARE GOOD
 - FLEXIBILITY IN CHOOSING PRESSURE SOLUTIONS IS QUITE ATTRACTIVE FOR CALCULATING HIGHER ORDER MODAL IMPEDANCES, AS WELL AS FOR DETERMINING THE RADIATION IMPEDANCE OF NON-PLANAR RADIATORS
-

FIGURE 11

In conclusion, it has been demonstrated that the Sherman formulation for calculating modal radiation impedances is sufficiently general to be utilized with any multi-mode transducer. The variational formulation for the radiated power, which does not require an exact formulation for the surface pressure, is readily recast into Sherman's form, and hence is useful for estimating these modal impedances. The comparison between variational and exact results in the case of baffled pistons and flexural disks is good in general, with poorer results for smaller values of ka and kd . Since no exact solution for the surface pressure is required, the variational method seems to be well-suited for calculating the radiation impedance of higher order modes of flexural disks, where the surface velocity will no longer be axisymmetric in the local coordinate system. A test of this would be to see if the radiated power, both for a single flexural disk as well as a pair, decreases significantly as one changes the velocity mode from the fundamental mode. In addition, impedances for non-planar radiators such as flexensionals could be calculated variationally, possibly using finite element methods to provide the *in-vacuo* surface velocities of the modes describing the transducer motion. The effects of phasing could also be included by allowing the transducer to have a complex surface velocity and pressure, allowing the effects of steering on radiation impedance to be ascertained. The resulting improvement in array modeling programs could potentially be dramatic.

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